Control of Moving Object Groups in Conflict Situation

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***Abstract*—The work is devoted to a review of research related to dynamic game problems with many participants on the opposing sides. The problems of collision avoidance and target interception are considered separately that corresponds to interests of opposing sides. When solving the problem of collision avoidance, the bypass method of Pontryagin – Mishchenko, methods of constant and alternated directions, method of invariant subspaces and recursive methods are used. The problems of group and successive pursuit as well as the problems of group interaction are studied on the basis of the method of resolving functions and Krasovskii' extreme aiming rule. The principles of shortest line and by-interval decomposition are formulated.**

***Keywords*—conflict-controlled process, collision avoidance, situation of incirclement, Caratheodori set, method of resolving functions, Pontryagin's condition, Apollonius circle.**

The global and the local problems of collision avoidance (avoiding meeting, evasion) with many participants on the opposing sides are considered. The goal in the global problem implies that a trajectory of conflict-controlled process should not hit the terminal set on all time-interval beginning from arbitrary initial states [1]. Specific feature of the global problem of collision avoidance consists in that for its solving it is suffice to construct the avoidance strategy only in some circumstance of the terminal set. This means that the conditions of the evader advantage can be given only on the terminal set because then they will hold by continuity in the circumstance as well [2], [3]. Unlike the global problem, in the local problem the initial states are given. On the basis of above mentioned methods, we obtained sufficient conditions for solvability of the nonlinear global collision avoidance problem in the form of minmax and maxmin inequalities, in the classes of piecewise-continuous epsilon-strategies and epsilon-counter-strategies. In so doing, crude and subtle cases, higher order conditions and conditions for escape from a group of pursuers are analyzed. Comparison of these conditions with the sufficient conditions in the form of inclusions for special set-valued mappings is provided. Note that in the crude case an advantage of the pursuer over each of the evader in projection on some two-dimensional subspace from the orthogonal complement to the terminal set (a linear subspace) is sufficient for escape. At the same time, in the subtle case it is sufficient to have such advantage only in projection on some two one-dimensional subspaces [3]. The scalar minmaxmin function, introduced by the author, plays a key role in the expression for advantage of the evader over each of the pursuer in the examples of escape of a single evader from a group of pursuers [2]. The method of invariant subspaces was brought about by the desire to ensure collision avoidance on the basis of minimal conditions on parameters of the conflict-controlled process [4]. In this method the advantage of the evader only on some one-dimensional subspace or equality in control resources in projection on a finite number of directions (Caratheodory set) is required. Therewith, rather rigid assumptions on invariance of the terminal set with respect to some matrices are imposed. Analysis of interaction of controlled object groups is restricted by the linear case [2], [5]. For the global problem of collision avoidance, under equal control resources of the players, sufficient conditions are obtained, which are expressed through the number of players on counteracting sides and properties of control domains, namely, their strict convexity and boundary smoothness. In the local problem, when formulating sufficient conditions, in addition to the parameters already mentioned, the mutual disposition of the players of the opposing sides plays an important role [2], [5]. In problems that implement the process of approaching of a trajectory (pursuit, target interception) of conflict-controlled process a terminal set of complex structure, a convenient tool is the method of resolving functions and, to a lesser extent, the extreme aiming rule. The method of resolving functions uses a completely natural accumulative principle, therewith the central role is played by the resolving function, meaningfully expressing the player's payoff at a given moment. In the course of the game, this function is accumulated in the integral sense up to a certain threshold value that fixes termination of the game. The method of resolving function, in particular, substantiates the law of parallel pursuit and approach along the beam, well known to designers of rocket and space technology. There is a number of the method schemes [5], [6], which reflect various aspects of a game problem and allow to terminate the game in classes of quasi- and stroboscopic strategies. All this can be realized under certain advantage of the first side – fulfillment of Pontryagin's condition or its modifications. Recently, the apparatus of upper and lower resolving functions has been developed on the basis of the shift function introduced by the author – an analogue of Pontryagin's selection in a situation where Pontryagin's condition does not hold. Also, the matrix resolving functions are introduced [9]. These innovations make it possible to expand the range of problems that can be solved, including with groups of participants [9-11]. The introduction of the above mentioned minmaxmin function stimulated the Pshenichnyi formalization of the environment situation in the group pursuit problem. It also contributed to obtaining the necessary and sufficient conditions for the game termination, which are conveniently checked in the case of a non-fixed time of the game termination [5], as well as in problems with phase constraints [5]. One of the most difficult game problems is the problem of successive pursuit [5]. Various schemes with fixed and non-fixed capture time of each of the evaders [5] are developed. Suggested constructions are especially efficient in the case of 'simple' motions with the program choice of order of capture. In this case, the pursuit strategy is the parallel approach and, therefore, the quality functional – the total capture time – depends only on controls of the evaders. Therewith, extremum of the functional is achieved on the constant controls of the evaders and the infinite-dimensional problem of maximization of the total pursuit time reduces to the finite-dimensional problem of conditional optimization. It should be noted that the strategy of parallel pursuit is closely connected with the Apollonius circle [5]. The shortest poly-line principle is stated to choose the order of the targets bypassing [5]. In the case of interaction of groups of controlled objects, the situation of environment is formalized for various capabilities of objects. For this situation, in general case the principle of interval decomposition is formulated, which is an iterative procedure and allows, on the basis of parallelization, to reduce the general problem of interaction of groups to the solution of simpler typical problems of target allocation, group and successive pursuit, including case of positional information [10-12]. Principle of interval decomposition consists in the following. Since there is a group of pursuers and a group of evaders then we divide them into subgroups (we carry out target distribution), each of which consists of either several pursuers and one evader, or one pursuer and several evaders. A one-on-one case can be assigned to any subgroup. Moreover, each of the pursuers can participate in only one subgroup. This division can be done in various ways: on the basis of some experience in planning operations or using the discrete optimization methods. In any case, instead of the most difficult problem of conflict counteraction between groups of controlled objects, we get several problems of group and successive pursuit. Thus, the process of interaction of groups is parallelized into independent sub-problems of group and successive pursuit. Let us fix the first moment when one of the sub-problems is solved. This means that at least one of the evaders is caught and, therefore, he or they can be excluded from further consideration, and the released pursuers can be used in other groups. Let us create a new partition of the groups of pursuers and remaining evaders into subgroups, each of which has either one pursuer or one evader and several opponents. Thus the process of optimization of groups of controlled objects is reduced to the iterative procedure and each iteration stage involves solving typical problems of target distribution, group and successive pursuit. The proposed method is effective for a large number of participants, because in this case the conflict-controlled processes proceeding in parallel are much simpler than the original problem of group interaction.

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