Coordinating Decentralized Consensus Control of Multy-Agent Networked Systems

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***Abstract*—Consensus control problem for multi-agent networked system is considered, when interconnection topology of agent's network is represented as directed weighted graph. A decentralized approach to consensus control design is proposed, including local stabilizing controllers design for each agent of networked system and consensus controllers design to ensure the conditions of agent's coordination. Local controllers design for individual agents is based on invariant ellipsoids approach and reduced to solving of linear matrix inequalities. Multi-agent system stability is performed by vector Lyapunov functions method which leads to stability analysis of reference system with low dimension.**

***Keywords*—consensus control; consensus protocol, invariant ellipsoids, linear matrix inequality, multi-agent system; networked system; vector Lyapunov function.**

INTRODUCTION

Coordinated consensus control is multi-agent system (MAS) is relevant and important direction in modern theory and practice of control [1, 2]. The application areas for consensus control, ensuring the coordination controlled dynamic objects in complex systems with network structures, are intelligent robotic control, vehicle and unmanned aircraft control, energy, transport and logistics systems and supply chain control.

The structure and communication topology of MAS is described by directed graph with nodes associated with controlled agents, and the edges represent information transfer. Control of MAS is carried out based on consensus protocol as linear feedback uses deviation of states of individual agents from the weighted average of the state vectors of its immediate neighbors [3]. Consensus problem in MAS is to align states of each individual agents to each other. So, the problem reduces to finding gain feedback matrices ensure stability of both controlled agents and MAS as a whole. The usual approach to solving this problem uses extended dynamic model of MAS with dynamics matrix based on Laplacian of communication topology graph and described by Kronecker products [4].

In this paper, we consider decentralized approach to consensus control in networked MAS, including local stabilizing controllers design for each agent of networked system and consensus controller design to ensure all agents' state vectors coordination. Control design based on invariant ellipsoids approach and reduced to linear matrix inequalities solving [5]. Stabilization of MAS problem is solved using low-dimension dynamic reference system constructed by vector Lyapunov functions (VLF) approach [6, 7].

PROBLEM STATEMENT

Consider MAS as a network of N agents, where dynamics of each i-th agent are described by discrete-time equation

$$x_i(k+1) = A_i\, x_i(k) + B_i\, u_i(k), \quad i = \overline{1,N}, \tag{1}$$

where for each i-th agent $x_i(k) \in \mathrm{R}^n$ is state vector and $u_i(k) \in \mathrm{R}^m$ is control input.

Communication topology of MAS represented by directed graph$G = (V, E)$, where $V = \{1, ..., N\}$is set of nodes (i.e., agents), and $E \subset V \times V$ is set of edges, reflecting information transfer between agents. Information, available to each agent$i$, is state vectors measurement of other agents directly connected with i in accordance with communication topology.

Control law, known as consensus protocol, taken as

$$u_i(k) = K_i x_i(k) + L_i v_i(k), \tag{2}$$

where $K_i \in \mathrm{R}^{m\times n}$ and $L_i \in \mathrm{R}^{m\times n}$ are feedback stabilizing and consensus gain matrices to be selected.

In turn, consensus term is formed as a linear combination of state vectors deviations of corresponding agents

$$v_i(k) = \frac{1}{S_i} \cdot \sum_{j=1}^{N} d_{ij}(x_i(k) - x_j(k)), \tag{3}$$

where $D = \{d_{ij}\} \in \mathrm{R}^{N\times N}$ is adjacency matrix associated with graph G; $S_i = \sum\limits_{j=1}^{N} d_{ij}$.

Control law (2) ensures consensus problem solution if

$$\|x_i(k) - x_j(k)\| \to 0 \text{ as } k \to \infty \;\; \forall i, j = \overline{1,N}, \tag{4}$$

that is conditions of coordinated behavior of individual agents.

Thus, consensus control problem is to choose control law matrices (2). At that, matrices K_i choice should ensure stabilization of closed-loop control subsystem for each agent, and matrices L_i ensure stability of controlled MAS altogether and fulfillment of consensus conditions (4).

Local agents stabilization

Dynamic equation of i-th agent with control (2) is

$$x_i(k+1) = A_i^f x_i(k) - \frac{1}{S_i}\sum_{j=1}^{N} d_{ij} B_i L_i\, x_j(k),$$
$$A_i^f = A_i + B_i(K_i + L_i). \quad (5)$$

Consider a set of quadratic Lyapunov functions:

$$V_i(k) = x_i^{\mathrm{T}}(k) P_i\, x_i(k), \quad P_i = P_i^{\mathrm{T}} \succ 0, \quad (6)$$

and a set of ellipsoids in system (5) state space

$$E_i(Q_i) = \{\, x_i \in \mathrm{R}^n : \;\; x_i^{\mathrm{T}}(k)\, Q_i^{-1} x_i(k) \;\le\; 1 \,\}, \quad (7)$$

which will be located inside the surface level of Lyapunov function (6) if $P_i = Q_i^{-1}$. Ellipsoid (7) called invariant for system (1) with control (2), if any trajectory of the system started in this ellipsoid remains there for any$k \ge 0$. Thus, the problem of i-th agent stabilization reduced to minimization of ellipsoids semi-axes squares sum, i.e. matrix Q trace.

Quadratic forms (6) will be a Lyapunov functions for any system (5) if and only if along system trajectory

$$\Delta V_i = V_i(k+1) - V_i(k) = s_i^{\mathrm{T}}(k) M_i\, s_i(k) < 0,$$
$$s_i(k) = \mathrm{col}\{x_1(k),\; x_2(k),\; ...,\; x_N(k)\}, \quad (8)$$

$$M_i = \begin{bmatrix} {A_i^f}^T P_i A_i^f - P_i & -\frac{d_{i1}}{S_i} {A_i^f}^T P_i L_i & \cdots & \cdots & -\frac{d_{iN}}{S_i} {A_i^f}^T P_i L_i \\ * & \frac{d_{i1}^2}{S_i^2} {L_i}^T {B_i}^T P_i B_i L_i & \cdots & \cdots & \frac{d_{i1} d_{iN}}{S_i^2} {L_i}^T {B_i}^T P_i B_i L_i \\ \vdots & \vdots & \ddots & \ddots & \vdots \\ * & * & \cdots & \cdots & \frac{d_{iN}^2}{S_i^2} {L_i}^T {B_i}^T P_i B_i L_i \end{bmatrix}.$$

Introduce the auxiliary variable matrices$Y_i = K_i Q_i$, $Z_i = L_i Q_i$. Because $Q_i \succ 0$, controller's matrices can be represented as$K_i = Y_i\, Q_i^{-1}$,$L_i = Z_i\, Q_i^{-1}$.

Constrained optimization problem solution

$$\mathrm{tr}\, Q_i \to \min,$$
$$\begin{bmatrix} Q_i & (A_i Q_i + B_i Y_i)^{\mathrm{T}} \\ A_i Q_i + B_i Y_i & Q_i \end{bmatrix} \succ 0,$$
$$Q_i \succ 0, \quad (9)$$

coincides with discrete matrix Lyapunov equation solution

$${A_i^f}^{\mathrm{T}} Q_i A_i^f - Q_i = -R_i, \;\; 0 \prec R_i = R_i^{\mathrm{T}} \in \mathrm{R}^{n \times n}. \quad (10)$$

In this way, solution of optimization problem using matrix trace criteria under the constraints in LMI form (9), defined controller, which stabilizes closed-loop agent local control subsystems. This approach allows reducing the problem of controller design to the solution of semi-definite programming problem, which can be found by known numerical methods.

Consensus control design

Introduce $x(k) = \left[x_1^{\mathrm{T}}(k),\; ...,\; x_N^{\mathrm{T}}(k)\right]^{\mathrm{T}} \in \mathrm{R}^{Nn}$ MAS extended state vector and consider vector Lyapunov function (VLF) [6, 7] and modular Lyapunov function for overall MAS

$$V_0\left(x(k)\right) = \sum_{i=0}^{N} p_{0i} \left|V_i\left(x_i(k)\right)\right| = P_0\, V\left(x(k)\right),$$
$$P_0 = [p_{01},\, ...,\, p_{0N}] \in \mathrm{R}^{1\times N}, \;\; p_{0i} > 0, \;\; i = \overline{1,N}. \quad (11)$$

Consider linear reference system

$$\upsilon(k+1) = \Lambda \upsilon(k), \quad \eta(k) = P_0 \upsilon(k),$$
$$V\left(x(k)\right) \le \upsilon(k), \quad V_0\left(x(k)\right) \le \eta(k), \quad (12)$$

where $\Lambda \in \mathrm{R}^{N\times N}$ – reference system dynamics matrix, $[\Lambda]_{ij} = \left(\mu_{ij}^{\max}\right)^{1/2}$,and $\mu_{ij}^{\max}$ is maximal roots values of quadratic forms pencils characteristic equations

$$\det\left({A_i^f}^{\mathrm{T}} P_i A_i^f - \mu_{ii} P_i\right) = 0, \quad i = \overline{1,N},$$
$$\det\left(L_i^{\mathrm{T}} B_i^{\mathrm{T}} P_i B_i L_i - \mu_{ij} P_j\right) = 0,$$
$$i,j = \overline{1,N}, \quad j \ne i. \quad (13)$$

Consensus gain matrices $L_i,\;\; i = \overline{1,N}$ must be chosen from reference system stability condition$\rho(\Lambda) < 1$, where $\rho(\Lambda)$ – spectral radius of reference system (11) dynamics matrix.

Conclusions

Decentralized approach to consensus control problem in MAS using LVF method leads to the stabilizing and consensus controllers design problem of reduced dimension, which is essential for network systems with complex structure. Further development of this approach is associated with taking into account control delays and decentralized solution of disturbances rejection problem and robust control of MAS as well as decentralized information fusion-based consensus state estimation for networked system.

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