# On real-time calculation of the rejected takeoff distance

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***Abstract*—A rejected takeoff is the situation when it is decided to cancel the takeoff of an airplane. The real rejected takeoff distances can differ from the pre-calculated values. The goal of our investigation is to obtain real-time estimations of the distance to full stop. Proposed algorithm takes into account different aircraft and environment parameters (thrust, velocity, etc.) before the moment of decision making. Thus, the decision about the rejected takeoff will be more reliable and safe.**

***Keywords*—rejected takeoff, braking distance, real-time algorithm, the Riccati equation.**

## INTRODUCTION

A rejected takeoff is the situation when it is decided to cancel the takeoff of an airplane. When a crucial obstacle to the takeoff occurs (e.g. engine failure), the crew compares the current velocity to the "decision-making velocity". Based on this velocity it is possible to estimate if the airplane still can safely stop within the available distance of the airport. If the current speed is less than the "decision-making velocity", the crew makes a decision to interrupt the takeoff; otherwise, it is necessary to continue the takeoff, avoiding overrun. This velocity and distances are calculated in advance using the "Aircraft Flight Manual"[1]. But the analysis of accidents shows that the real rejected takeoff distances can differ from the pre-calculated values [2, 3]. The goal of our investigation is to obtain real-time estimations of the distance to full stop. The authors previously proposed game approach to control aircraft on its takeoff [4].

## REAL-TIME ALGORITHM

The aircraft motion on a runway can be described by differential equations [5]

$$m\frac{dv}{dt} = P\cos\alpha_P - Q - mg\sin\theta + f(Y - mg)\cos\theta, \quad (1)$$

$$\frac{dl}{dt} = v,\ l(0) = 0,\ v(0) = 0, \quad (2)$$

where $t$ – time, $v$ – velocity, $l$ – distance on the airport runway, $m$ – mass, $P$ – thrust of all aircraft engines, $\alpha_P$ – aircraft engine angle of attack, $Q = c_x\frac{\rho v^2}{2}S$ – aircraft drag force, $Y = c_y\frac{\rho v^2}{2}S$ – aircraft lift, $c_x$ – aircraft drag coefficient, $c_y$ – aircraft lift coefficient, $S$ – relevant surface area, $\rho$ – air density, $g$ – acceleration of gravity, $\theta$ – runway angle, $f$ – friction coefficient.

Equation (1) is a Riccati equation

$$\frac{dv}{dt} = -\Lambda v^2 + G(t), \quad (3)$$

$$\Lambda = \frac{\rho S}{2m}(c_x - f\cdot c_y),$$

$$G = \frac{\cos\alpha_P}{m}P - g(\sin\theta - f\cos\theta),$$

where in (3) both $\Lambda$ and $G$ depend on aircraft and environment parameters (aircraft mass, friction coefficient, air density, lift and drag coefficients, etc.), but only $G$ depends on the thrust.

The real-time algorithm for calculation of the interrupted flight distance was proposed in [6]. It uses step interpolation of the jet thrust. Two intervals of the thrust change (when the thrust decreases from the maximum to zero and then increases to the nominal reversal thrust) can be substituted by three intervals of the constant thrust (full flight thrust, zero thrust, reverse thrust). With this interpolation, the Riccati equation has three different solutions, corresponding to three different constant thrusts. Using this approach, it is possible to obtain explicit solutions of the motion equation, that can be used in real-time calculations. For example, on time intervals with constant positive thrust, we can calculate the distance and velocity at the end of this time interval:

$$\Delta l_1 = \frac{1}{\Lambda}\ln\left[\frac{\Lambda v_0}{\gamma}sh(\gamma\Delta_1) + ch(\gamma\Delta_1)\right], \quad (4)$$

$$v_1 = e^{-\Lambda\Delta l_1}\left[v_0 ch(\gamma\Delta_1) + \frac{\gamma}{\Lambda}sh(\gamma\Delta_1)\right], \quad (5)$$

where $\gamma = \sqrt{\Lambda G}\ (\Lambda G > 0)$, $\Delta_1 = t_1 - t_0$ is the time interval that starts at the moment $t_0$ and finishers at the moment $t_1$, with constant positive thrust, $\Delta l_1 = l(t_1) - l(t_0)$ is the distance travelled by the aircraft during this time interval, $v_0 = v(t_0)$, $v_1 = v(t_1)$. Similarly, the distance of the last time interval $\Delta l_3 = l(t_3) - l(t_2)$, up to the full stop at the moment $t_3$, when the aircraft runs with maximum reverse thrust, can be calculated as

$$\Delta l_3 = \frac{1}{2\Lambda}\ln\left[1 - \frac{\Lambda v_2^2}{G}\right], \quad (6)$$

where $v_2 = v(t_2)$ is the velocity at the beginning of this time interval, and $\Lambda G < 0$ during this time interval.

The real process of any rejected takeoff is more complicated than described in [6]. After decision to interrupt the takeoff is chosen, in addition to thrust change to reversal, the crew changes some aerodynamic parameters of the aircraft using spoilers and other wing controls, and applies wheel brakes. Also, reverse thrust depends on the current velocity, and, moreover, when the velocity decreases below a particular value, the reverse thrust is changed to the low reverse thrust. So, unlike the model used in [6], equation parameters depend not only on time, but also on velocity. Our investigations show that it is still possible to use step interpolation of the thrust, brake coefficient and some other parameters to obtain correct estimation of the distance to the stop. Using explicit solutions of the Riccati equation, we can obtain explicit formula connecting time, velocity, and distance in the motion equations. Then we use these dependencies to estimate final stop distance.

## Emulation

To verify the proposed algorithms, a system for computer emulation was created. With this system, we can compare the rejected takeoff distance obtained using the proposed algorithm with the distance calculated from full motion model using the numerical solution of the differential equations. The system allows to vary numerous aircraft parameters (more than 30) and initial states. These parameters are based on values for AN-124 aircraft. The emulation shows that we can use only 8 time steps when aircraft parameters are changed significantly. Fig. 1 shows the emulation of the airplane thrust. The thick line (Real) shows how the thrust changes from the moment when the decision to interrupt the takeoff is taken to the moment of full stop. This thrust depends on time and aircraft velocity. The thin line (Emulated) depicts emulated values of the thrust. Fig. 2 shows the emulation of the distance run by aircraft. The thick line (Real) shows the aircraft distance on the runway. This distance is calculated using the numerical solution of the differential equations with the thrust shown on Fig. 1, and with other parameters that depend on time and velocity (lift, drag, and friction coefficients, mass, etc.). The thin line (Emulated) shows the constant emulated values for the distance, that are calculated using constant estimations of the aircraft parameters.

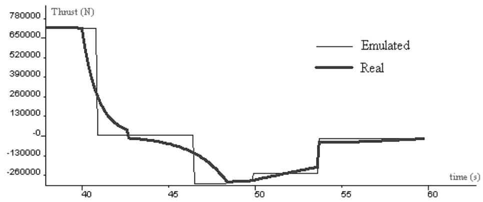


Fig. 1. Thrust emulation

The results of emulation show that the proposed algorithm can estimate the distance to the full stop with the accuracy of several meters. This distance is also in good agreement with the values obtained from flight nomograms [1].

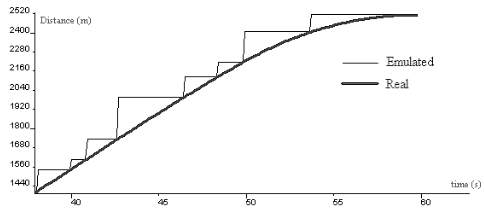


Fig. 2. Rejected takeoff distance emulation

## Conclusions

The algorithm proposed to calculate rejected takeoff distance uses step interpolation of aircraft thrust and other parameters. This interpolation allows to obtain explicit solutions of the Riccati equation that describes the aircraft motion. With this algorithm, it is possible to perform calculations quickly, in real time. Algorithm takes into account aircraft parameter just in the moment of emergency. With this method, the accuracy of the rejected takeoff distance estimation is better comparing to the pre-calculated values. So the decision about the takeoff interrupt can be more accurate and safe.

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