Principle of Time Stretching in Game Problems of Dynamics

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***Abstract*—We consider the dynamic games of pursuit which are described by a system of general form that encompasses a wide range of functional-differential systems. On the basis of equivalence of the game of pursuit with delay of information to the game with complete information with the changed dynamics and the terminal set, the principle of time stretching is developed to analyze the games, for which Pontryagin's condition , lying at the heart of all direct methods of pursuit, does not hold. Investigation is carried out in the frames of the first direct method and the method of resolving functions.**

***Keywords*—time stretching, Pontryagin's condition, terminal set, time-variable information delay, terminal set, method of resolving function, soft meeting**

There exists a wide range of mechanical, economical and biological processes which can be described by dynamic systems of various kinds, in particular, by the ordinary differential, difference, difference-differential, integral, integro-differential, partial differential and fractional equations, as well as by impulse systems, depending on the process nature. Any disturbance, counteraction or inaccuracy readily leads to game situation [1]. The deciding factor in study of dynamic games is availability of information on current state of the process, its prehistory or various kind counterpart's discrimination. A number of fundamental methods are developed in the mathematical theory of control in condition of conflict and uncertainty. Most of them deal with the dynamics described by ordinary differential equations. The desire to find optimal strategies for the behavior of the opposing sides in a conflict situation invariably leads to the ideology of dynamic programming that lies in the foundation of the R. Isaacs method [2], which is based on the main Hamilton-Jacobi-Bellman-Isaacs equation. In the terrns of relationships for sets, this ideology is well expressed in Pontryagin's method of alternated integral [3] and its outgrowth – Pshenichnyi' method of semi-group operators [4]. Krasovskii' theorems on alternative [5] should be assigned to this group of results.

Sometimes the goal in practice is to achieve the desired result without worrying about optimality. This seemingly more justified goal underlies the rule of the extreme aiming rule of Krasovskii [5], the first direct method of Pontryagin [2] and the method of resolving functions together with its modifications [6–8]. In real systems information often arrives with delay in time. It is shown that the dynamic game of pursuit with variable information delay is equivalent to certain perfect-information game with the changed dynamics and the terminal set. It was first proved for the linear differential games with constant delay of information, then for the case of variable information delay [9]. This effect of information delay opened up possibilities for application of classic methods to analyze the games with delay of information [9, 10].

The central condition for the realization of the pursuit process based on a measurable choice in the first direct method of Pontryagin [2] and the original form of the method of resolving functions [7] is Pontryagin's condition. This condition is expressed in terms of set-valued mappings and ensures overwhelming advantage of the pursuing side. However, it does not hold in most cases of conflict withstanding. This is the case, for example, for objects with different inertia and in the problem of soft meeting [7], [11]. There exist many ways of modification of Pontryagin's condition. They are outlined in [7]. In particular, one of them consists in inclusion into the condition of the solid part of cylindrical terminal set, another one – to suppress the enemy's resource with subsequent repayment of the debt that leads to two relations in the general condition. Analysis of Pontryagin's condition performed by Nikolskij [11] significantly advanced its understanding and was a contributory factor to this condition modification [12], prescribing construction of the pursuer control on the basis of the evader one in the past. Notice that the result of analysis, provided in [11] of the model example contained mistake that was eliminated in [13].

Establishment of close relation of the modified condition with the passage from the original game with perfect information to an auxiliary one with delayed information [13-15] gave impetus to the development of efficient approach (the principle of time stretching) to solving complicated games of pursuit, namely, those for which Pontryagin's condition does not hold [15–16]. Here we consider the dynamic games of pursuit which are described by a system of general form which encompasses a wide range of functional-differential systems [17].

This paper is devoted to one of the way of modification of generalized Potryagin's condition for such games. It consists in introduction of special function of time – the function of time stretching. The goal of introducing this function is to provide fulfillment of the modified Pontryagin's condition and to terminate the

game at the finite time. We name it by the principle of time stretching. It should be noted that the principle of time stretching is closely related with the game problems with variable delay of information [18, 19]. The gist of the time stretching principle consists in artificial worsening the availability of information on the current evader control to the pursuer. It is assumed that that information about behavior of the evader arrives to the pursuer with the time delay. In fact, the transition is made from the original game with complete information to the game with the same dynamics and the terminal set, yet with special kind information delay. This delay is a function of time, decreasing as the game trajectory approaches the terminal set and vanishing as it hits the target. The central idea of the time stretching principle consists in introduction of certain function, called the time stretching function, in which terms the time delay is expressed in explicit form. The obtained game with delayed information is analyzed on the basis of its equivalence to the perfect-information game with the changed dynamics. An important point is that Pontryagin's condition for the latter game involves the time stretching function. The time stretching principle makes it feasible to deduce sufficient condition fo the game termination.

The efficiency of results are supported by the examples of conflict-controlled systems with exact capture in geometric coordinates and soft meeting (simultaneous coincidence of geometric coordinates and velocities) of controlled objects [15, 16]. The time stretching principle proved its efficiency in solving the problems of soft meeting in various cases of second-order dynamics, for which formula for the time stretching function is deduced in explicit form, in their number, in the case of oscillatory dynamics [16] and different-kind dynamics of the pursuer and the evader [18]. Simple conditions on the games parameters insuring feasibility of the pursuit termination are deduced. The geometric-descriptive situation of finding 'tracks' of the evader is studied in detail, that provides realization of the time stretching principle by the way of the pursuer following the evader's trajectory with delay in time [15]. The time stretching principle is applied to the dynamic games of pursuit, described by a system of general form [18, 19]. The result of investigation is specified for the integral- differential game of pursuit [20]. To this end, we infer solution to the integro-differential system in the Cauchy form. In so doing, the method of successive approximations is used to find solution of the Volterra second-order integro-differential equation. To support the suggested technique, simple example of integro-differential pursuit game is examined in detail.

In line with this direction, a linear differential game of pursuit is considered when the opposing sides use impulse controls [21]. Processes with impulse control arise, in particular, in space research when considering the problems of the traveling salesman type associated with target allocation and control of moving objects. A method for solving the problem is proposed, based on the use of the time stretching function, which makes its possible to deduce sufficient conditions for the game termination in a finite guaranteed time. The results are illustrated by a model example of a game with the dynamics of mathematical pendulum. The principle of time stretching is also applied to the method of resolving functions and its modifications related with introduction of the shift function and the upper and the lower resolving functions. This method makes it possible to terminate the game using quasi- and stroboscopic strategies constructed on the basis of special set-valued mappings and properties of their selections.

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